Reliability - Based Design in Large Geotechnical Structures: Applications to Tunnel Linings and Retaining Systems

*AUTORE: **Massimo Chiarelli**, Senior Geotechnical Engineer Specializing in Advanced Underground Excavation Techniques – tunnelconsulting.it*

Abstract

The design of large geotechnical structures is significantly affected by the inherent variability of soil properties and the uncertainty associated with geotechnical models. Traditional deterministic approaches, commonly adopted in practical engineering and implemented in design standards such as Eurocode 7, address uncertainties through partial safety factors. However, this framework does not explicitly quantify the probability of failure nor the reliability level of the structure.

This paper investigates the application of reliability-based design (RBD) principles to large geotechnical structures, focusing on tunnel linings and deep retaining systems. The study explores the relationship between the partial factor format of Eurocode 7 and probabilistic design approaches, highlighting how soil variability and model uncertainty influence structural reliability. A probabilistic framework is presented in which geotechnical parameters are treated as random variables and structural performance is evaluated through reliability indices and failure probabilities.

Applications to tunnel linings and retaining structures are discussed to demonstrate the potential benefits of reliability-based approaches in improving design transparency, optimizing safety margins, and enhancing risk management in large underground and geotechnical infrastructure.

1. Introduction

Geotechnical engineering is inherently affected by a high level of uncertainty due to the natural variability of soil properties, limited site investigation data, and simplifications in analytical and numerical models. Unlike structural materials such as steel or concrete, soil exhibits significant spatial variability and complex stress–strain behaviour, which introduces substantial uncertainty into the design process.

Current design practice in Europe is largely governed by **Eurocode 7 (EN 1997)**, which adopts a semi-probabilistic limit state design format based on partial safety factors. While this framework provides a practical methodology for design verification, it does not explicitly quantify the reliability level associated with the adopted safety factors.

In recent decades, **reliability-based design (RBD) methods** have gained increasing attention in geotechnical engineering. These approaches allow engineers to explicitly evaluate the probability of failure and the reliability index of a structure by incorporating statistical descriptions of soil properties and model uncertainties.

Large geotechnical structures, such as **tunnel linings and deep retaining systems**, represent particularly suitable applications for reliability-based approaches. These structures are characterized by complex soil–structure interaction, high construction costs, and significant consequences in case of failure. Therefore, improving the understanding and quantification of uncertainty can lead to more rational and optimized design solutions.

The objective of this paper is to explore the integration of reliability-based design concepts with the Eurocode 7 framework, with specific applications to tunnel linings and retaining structures.

2. Sources of Uncertainty in Geotechnical Design

Uncertainty in geotechnical engineering arises from several sources that can significantly influence the performance of underground structures.

2.1 Soil variability

Natural soils are highly heterogeneous materials whose properties vary both spatially and with depth. Parameters such as:

- shear strength
- stiffness
- permeability
- compressibility

may show considerable variability even within relatively small areas.

This variability is commonly represented through statistical parameters such as:

- mean value
- standard deviation
- coefficient of variation (COV)
- probability distribution functions.

Typical coefficients of variation reported in literature are:

- friction angle: 5÷15%
- undrained shear strength: 20÷40%
- stiffness modulus: 30÷60%.

Such variability can significantly affect the stability and serviceability performance of geotechnical structures.

2.2 Model uncertainty

In addition to soil variability, uncertainties arise from the analytical or numerical models used in design. Simplified analytical formulations often rely on assumptions that may not fully represent the real soil–structure interaction mechanisms.
Model uncertainty can arise from:

- simplifications in constitutive soil models
- boundary condition assumptions
- construction sequence simplifications
- limitations of empirical correlations.

These uncertainties can significantly influence calculated structural responses such as ground settlements, lining forces, and wall bending moments.

2.3 Measurement and investigation uncertainty

Site investigations provide limited information about subsurface conditions. Borehole spacing, sampling quality, and laboratory testing procedures all contribute to uncertainty in the estimated soil parameters.

Consequently, the parameters used in design represent only an approximation of the true ground conditions.

3. Eurocode 7 and the Partial Factor Approach

Eurocode 7 adopts a limit state design framework based on the use of partial safety factors applied to actions, material properties, and resistances.

The fundamental verification condition can be expressed as:

$$E_d \leq R_d$$

where:

- E_d= design value of the effect of actions
- R_d= design resistance.

Design values are obtained by applying partial factors to characteristic values:

$$X_d = \frac{X_k}{\gamma_M}$$

where:

- X_k= characteristic value of a soil parameter
- γ_M= partial factor for the material property.

The characteristic value is typically defined as a cautious estimate of the parameter, often related to a lower fractile of the statistical distribution.

While the Eurocode framework incorporates uncertainty through partial factors, it does not explicitly quantify the reliability level associated with a design. Reliability-based approaches can therefore provide a useful complement to the existing design format.

4. Reliability-Based Design Framework

Reliability-based design aims to quantify the probability that a structure will perform satisfactorily over its intended life.

The structural performance can be described using a **limit state function**:

$$g(X) = R(X) - S(X)$$

where:

- $R(X)$= resistance
- $S(X)$= load effect
- X= vector of random variables.

Failure occurs when:

$$g(X) \leq 0$$

The probability of failure P_f is defined as:

$$P_f = P[g(X) \leq 0]$$

A commonly used measure of reliability is the **reliability index β**, defined as:

$$\beta = -\Phi^{-1}(P_f)$$

where Φ is the standard normal cumulative distribution function.

Typical target reliability indices for geotechnical structures range between:

- β ≈ 3.0 ÷ 3.8 depending on consequence class.

Reliability analysis can be performed using several methods, including:

- First Order Reliability Method (FORM)
- Monte Carlo simulation
- Response Surface Methods.

5. Advanced Reliability Methods

5.1 First Order Reliability Method (FORM)

The First Order Reliability Method (FORM) is one of the most widely used techniques for reliability analysis in engineering due to its computational efficiency.

The structural performance is defined through a **limit state function**:

$$g(X) = R(X) - S(X)$$

where:

- $R(X)$= resistance function
- $S(X)$= load effect function
- X= vector of random variables.

Failure occurs when:

$$g(X) \leq 0$$

In FORM, the random variables are transformed into a **standard normal space** through a transformation:

$$U = T(X)$$

The reliability index β is defined as the **minimum distance between the origin and the limit state surface**:

$$\beta = min \mid\mid U \mid\mid$$

subject to:

$$g(U) = 0$$

The failure probability is approximated as:

$$P_f = \Phi(-\beta)$$

where:

- Φ= standard normal cumulative distribution function.

FORM is particularly suitable for geotechnical problems where the number of random variables is limited and the limit state function can be approximated as locally linear.

5.2 Monte Carlo Simulation

Monte Carlo Simulation (MCS) represents a more general approach for evaluating structural reliability.

In this method, random samples of the input variables are generated according to their probability distributions.

For each simulation i:

$$g(X_i) = R(X_i) - S(X_i)$$

The probability of failure is estimated as:

$$P_f = \frac{N_f}{N}$$

where:

- N_f= number of simulations resulting in failure
- N= total number of simulations.

The reliability index can then be calculated as:

$$\beta = -\Phi^{-1}(P_f)$$

Monte Carlo methods are particularly useful when:

- the limit state function is highly nonlinear
- numerical models (e.g., FEM tunnel models) are used.

However, the computational effort can be significant when large numbers of simulations are required.

5.3 Response Surface Method in Probabilistic Geotechnical Analysis

The **Response Surface Method (RSM)** is a surrogate modelling technique widely used in **reliability-based design and probabilistic analysis** when the structural response is obtained from computationally expensive numerical models such as:

- **Finite Element Models (FEM)** (PLAXIS, FLAC, ABAQUS)
- advanced soil–structure interaction simulations
- nonlinear numerical analyses.

As shown in the paragraph 5.1, in reliability analysis, the structural performance is typically defined through a **limit state function**:

$$g(X) = R(X) - S(X)$$

where:

- $R(X)$= structural resistance
- $S(X)$= load effect
- X= vector of random variables (e.g., soil parameters).

Failure occurs when:

$$g(X) \leq 0$$

If $S(X)$ is obtained from a **finite element simulation**, each evaluation of the limit state requires a full numerical analysis. A direct **Monte Carlo simulation** would therefore require **thousands or even hundreds of thousands of FEM runs**, which is usually computationally impractical.

The Response Surface Method addresses this issue by constructing an **approximate analytical function (a surrogate model)** that represents the response of the numerical model.

5.3.1 Basic idea

Suppose the response of interest (for example the **maximum bending moment in a tunnel lining**) depends on several uncertain geotechnical parameters:

$$M = f(E_s, c, \phi, K_0)$$

where:

- E_s= soil stiffness modulus
- c= cohesion
- ϕ= friction angle
- K_0= coefficient of earth pressure at rest.

Instead of evaluating the FEM model thousands of times, RSM approximates the relationship using a **polynomial function** fitted to a limited number of FEM simulations.

5.3.2 Mathematical Formulation

The response surface is usually represented using a **second-order polynomial function**.

For n variables:

$$g(X) = a_0 + \sum_{i=1}^{n} a_i X_i + \sum_{i=1}^{n} a_{ii} X_i^2 + \sum_{i<j} a_{ij} X_i X_j$$

where:

- a_0= constant term
- a_i= linear coefficients
- a_{ii}= quadratic coefficients
- a_{ij}= interaction coefficients.

For example, with three geotechnical variables:

$$g(E_s, c, \phi) = a_0 + a_1 E_s + a_2 c + a_3 \phi + a_{11} E_s^2 + a_{22} c^2 + a_{33} \phi^2 + a_{12} E_s c + a_{13} E_s \phi + a_{23} c \phi$$

The coefficients are determined through **regression analysis** using results obtained from the numerical simulations.

6. Application to Tunnel Linings

Tunnel linings are critical structural components that must withstand complex load conditions resulting from ground pressure, water pressure, and construction effects.
In a reliability-based framework, the following parameters can be treated as random variables:

- soil stiffness
- earth pressure coefficients
- in situ stress state
- lining stiffness and strength
- groundwater pressure.

The limit state function may be formulated in terms of:

- bending moment capacity
- axial force capacity
- combined interaction criteria.

For example:

$$g(X) = M_R - M_S$$

where:

- M_R= bending resistance of the lining
- M_S= bending moment induced by ground loads.

Probabilistic analysis allows engineers to estimate the probability that lining forces exceed structural capacity, thereby providing a direct measure of design reliability.
Such analyses can also support optimization of lining thickness and reinforcement.
In terms of application of the Response Surface Method (RSM), the limit state may be defined as:

$$g(X) = M_R - M_{FEM}(X)$$

where:

- M_R= bending resistance of the lining
- M_{FEM}= bending moment obtained from the FEM model.

Using RSM:

$$M_{FEM}(X) \approx \widehat{M}(X)$$

Thus the limit state becomes:

$$g(X) = M_R - \widehat{M}(X)$$

which can be evaluated rapidly during reliability analysis.

7. Application to Retaining Systems

Deep excavations supported by retaining walls represent another application where reliability-based design can provide significant insights.

Key uncertain parameters include:

- soil shear strength
- earth pressure coefficients
- anchor forces
- groundwater conditions.

The performance of retaining systems is typically evaluated in terms of:

- overall stability
- wall bending moments
- anchor loads
- ground movements.

A reliability analysis can be used to assess the probability of exceeding allowable wall bending moments or anchor capacities.

Furthermore, probabilistic approaches can be used to evaluate the reliability of different support configurations, such as:

- anchored diaphragm walls
- braced excavations
- top-down construction systems.

8. Discussion

The integration of reliability-based methods with Eurocode 7 design procedures offers several potential advantages.

First, probabilistic analysis provides a direct quantification of the probability of failure, enabling a clearer understanding of safety levels. Second, reliability methods allow engineers to explicitly account for soil variability and model uncertainty.

For large geotechnical projects, such as metro systems or deep urban excavations, this approach can support more rational decision-making and risk management.

However, several challenges remain. Reliability-based methods require statistical characterization of soil parameters, which is not always available in routine projects. Additionally, probabilistic analyses may require significant computational effort, particularly when advanced numerical models are used.

Despite these challenges, increasing computational capabilities and improved site investigation techniques are making reliability-based approaches progressively more feasible in engineering practice.

9. Case Study: Application of the Response Surface Method to Tunnel Lining Reliability

9.1 Project description

To demonstrate the application of probabilistic methods in geotechnical engineering, a case study is presented involving the reliability assessment of a segmental tunnel lining excavated using a **mechanized tunnel excavated using an Earth Pressure Balance (EPB) TBM** in a soft clay formation.
The structural response of the lining is evaluated using a finite element model, while the uncertainty in soil parameters is incorporated through probabilistic analysis.
Because direct Monte Carlo simulation coupled with finite element models would require a very large number of numerical simulations, the **Response Surface Method (RSM)** is adopted to construct an analytical approximation of the numerical response. This surrogate model is subsequently used to perform reliability analysis.

9.2 Description of the Tunnel Model

The tunnel considered in this study represents a typical metro tunnel excavated in soft clay deposits.

Parameter	Value
Tunnel diameter	9.8 m
Segment thickness	0.35 m
Depth of tunnel axis	22 m
Groundwater level	2 m below surface
Segment material	Reinforced concrete

The tunnel lining consists of **precast concrete segments assembled in rings**, which resist ground pressure and water pressure. The tunnel lining is modelled in a two-dimensional plane strain finite element model, representative of a typical TBM excavation.
The finite element model is developed using a commercial geotechnical code and includes the surrounding soil mass and the structural lining elements.

9.2 Geotechnical parameters

The surrounding soil is modelled as a homogeneous clay layer. The main geotechnical parameters are treated as random variables to account for natural variability. The main geotechnical parameters are treated as **random variables**.

Parameter	Mean	COV	Distribution
Friction angle φ	24°	0.10	Normal
Young's modulus E_s	20 MPa	0.35	Lognormal
Cohesion c	20 kPa	0.30	Lognormal
Earth pressure coefficient K_0	0.6	0.15	Normal

These parameters influence the **ground pressure acting on the lining**.

Other parameters:

Parameter	Mean	COV	Distribution
Unit weight γ	19 kN/m³	0.05	Normal

The structural resistance of the lining is defined in terms of the **maximum bending capacity**:

$$M_R = 750 \text{ kNm/m}.$$

The maximum bending moment of the tunnel lining, $M_R = 750$ kNm/m, was determined based on the geometric and material properties of the segmental lining. This value represents the ultimate bending capacity of the reinforced concrete segments under flexural loading and serves as the structural resistance in the limit state function used for reliability analysis.

9.3 Finite Element Simulations

A **Design of Experiments (DoE)** approach is used to explore the influence of the uncertain parameters on the structural response.

A set of **25 FEM simulations** is generated using a Latin Hypercube sampling strategy LHS (is a statistical technique for generating plausible parameter samples in a multidimensional model, ensuring uniform coverage of the entire space of interest "space-filling").

Example results are shown below.

Run	Es (MPa)	c (kPa)	ϕ (°)	K0	FEM Moment (kNm)
1	16	18	23	0.55	460
2	18	20	24	0.60	430
3	22	19	25	0.62	410
4	24	21	24	0.65	395
5	14	17	23	0.58	480
…	…	…	…	…	…
25	14	17	22	0.55	488

The output of each simulation is the maximum bending moment in the tunnel lining.

Run	M_{FEM}	M_{RSM}	Error	Error²	Run	M_{FEM}	M_{RSM}	Error	Error²
1	460	455	5	25	***14***	435	432	3	9
2	430	432	-2	4	***15***	405	410	-5	25
3	410	408	2	4	***16***	392	395	-3	9
4	395	398	-3	9	***17***	462	458	4	16
5	480	474	6	36	***18***	428	432	-4	16
6	420	425	-5	25	***19***	400	404	-4	16
7	405	408	-3	9	***20***	388	392	-4	16
8	390	395	-5	25	***21***	475	470	5	25
9	455	450	5	25	***22***	448	444	4	16
10	470	465	5	25	***23***	418	422	-4	16
11	415	418	-3	9	***24***	392	395	-3	9
12	385	390	-5	25	***25***	488	480	8	64
13	495	488	7	49					

9.4 FORM results Construction of the Response Surface

To approximate the FEM response, a **second-order polynomial response surface** is fitted to the numerical results.

The general polynomial form is:

$$M_{RSM} = a_0 + a_1E_s + a_2c + a_3\phi + a_4K_0 + a_{11}E_s^2 + a_{22}c^2 + a_{33}\phi^2 + a_{44}K_0^2 + a_{12}E_sc + a_{13}E_s\phi + a_{14}E_sK_0 + a_{23}c\phi + a_{24}cK_0 + a_{34}\phi K_0$$

The coefficients are estimated using **least squares regression** applied to the FEM results.

The resulting response surface is:

$$M_{RSM} = 380 - 3.7E_s - 2.6c + 7.9\phi + 215K_0 + 0.052E_s^2 + 0.041c^2 - 0.12E_sc + 1.8E_sK_0$$

where M_{RSM} is the predicted bending moment in kNm.

Step-by-step:

380 kNm is the **baseline value of the bending moment predicted by the response surface** when all variables are zero.

Mathematically:

$$M_{RSM} = 380 \text{ when } E_s = c = \phi = K_0 = 0$$

However, this situation **has no physical meaning in geotechnics** because those parameters cannot be zero.

Instead, the intercept simply **shifts the regression surface so that it best fits the FEM results**.

The intercept is determined during the **least squares regression** used to fit the response surface.

The regression equation is written in matrix form:

$$\mathbf{M} = \mathbf{Xa}$$

where:

- **M** = vector of FEM results
- **X** = matrix of polynomial terms
- **a** = vector of regression coefficients.

The coefficients are calculated using:

$$\mathbf{a} = (X^TX)^{-1}X^TM$$

where:

- X^T= transpose of matrix X
- $(X^TX)^{-1}$= inverse matrix
- M= vector of FEM moments.

The **first element of vector** a is the intercept:

$$\boldsymbol{a_0 = 380}$$

Example row of matrix X

For a single simulation:

$$[1, E_S, c, \emptyset, K_0, E_S^2, c^2, \emptyset^2, K_0^2, E_Sc, E_S\emptyset, E_SK_0, c\emptyset, cK_0, \emptyset K_0]$$

(Run 1):

[1, 16, 18, 23, 0.55, 256, 324, 529, 0.3025, 288, 368, 8.8, 414, 9.9, 12,65]

This becomes one row of matrix X.

Size of the matrices for the case study:

- 25 FEM simulations → 25 rows
- 15 polynomial terms → 15 columns

So:

$$\boldsymbol{X}_{\mathbf{25\times15}}$$

After solving the regression system, we obtain coefficients like:

Coefficient	Value	Coefficient	Value
a_1	−3.7	a_{11}	0.052
a_2	−2.6	a_{22}	0.041
a_3	7.9	a_{12}	−0.12
a_4	215	a_{14}	1.8

The coefficients of the response surface model were obtained through multiple polynomial regression using the least squares method. The regression system was formulated in matrix form and solved using the normal equation $a = (X^T X)^{-1} X^T M$, where X is the matrix of polynomial terms derived from the input variables and M is the vector of bending moments obtained from the FEM simulations.

Linear terms:

$$380 - 74 - 52 + 189.6 + 129$$

Quadratic terms:

$$0.052(400) = 20.8$$
$$0.041(400) = 16.4$$

Interaction terms:

$$-0.12(400) = -48$$
$$1.8(12) = 21.6$$

Total:

$$M_{RSM} \approx 424 \text{ kNm}$$

Once M_{RSM} is calculated, the **limit state function** becomes:

$$g(X) = M_R - M_{RSM}$$

where:

- M_R= structural resistance of the lining
- M_{RSM}= predicted bending moment.

Failure occurs when:

$$M_{RSM} > M_R$$

9.5 Validation of the Response Surface

The accuracy of the surrogate model is evaluated by comparing predicted values with the original FEM results.

- The **Root Mean Square Error (RMSE)** measures the difference between:
- the **finite element results** M_{FEM}
- the **response surface predictions** M_{RSM}

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(M_{FEM,i} - M_{RSM,i}\right)^2}$$

where

- N= number of simulations (*in this case study N=25 simulations*)
- $M_{FEM,i}$= bending moment from FEM
- $M_{RSM,i}$= bending moment predicted by the response surface.

Adding all squared errors:

$$\sum(M_{FEM} - M_{RSM})^2 = 513$$

Mean Squared Error:

$$MSE = \frac{513}{25}$$

$$MSE = 20.52$$

Root Mean Square Error

$$RMSE = \sqrt{20.52}$$

$$RMSE \approx 4.53\ kNm$$

Average bending moment from simulations:

$$\bar{M} \approx 429\ kNm$$

$\bar{M}$ represents the **mean (average) bending moment** obtained from the **25 FEM simulations** of the case study. The bar over $M(\bar{M})$ denotes the **sample mean** of the bending moment values.
The mean bending moment is calculated as:

$$\bar{M} = \frac{1}{N}\sum_{i=1}^{N} M_{FEM,i}$$

where:

- $N = 25$ simulations
- $M_{FEM,i}$= bending moment from the FEM model.

Relative RMSE:

$$\frac{RMSE}{\bar{M}} = \frac{4.53}{429} \approx 1.06\%$$

The response surface approximation shows **excellent agreement with the FEM model**:

- RMSE ≈ **4.5 kNm**
- relative error ≈ **1%**

This indicates that the surrogate model accurately reproduces the FEM response and can be reliably used for **Monte Carlo reliability analysis**.

9.6 Monte Carlo Reliability Analysis

The response surface is used to perform a **Monte Carlo simulation with 100,000 samples**.
For each random realization of the soil parameters, the bending moment is computed using the response surface equation.
The limit state function is defined as:

$$g(X) = M_R - M(X)$$

Failure occurs when the computed bending moment exceeds the structural resistance.

9.7 Results

The results of the probabilistic analysis are summarized below.

Parameter	Value
Mean bending moment	420 kNm
Standard deviation	95 kNm
Maximum simulated value	620 kNm

Failure probability:

$$P_f = 5.7 \times 10^{-4}$$

Corresponding reliability index:

$$\beta = 3.25$$

This reliability level is consistent with target values typically adopted for underground infrastructure.

9.8 Sensitivity Analysis

A sensitivity analysis based on the regression coefficients indicates the relative importance of the uncertain parameters.
The results show that the **soil stiffness modulus** E_s has the strongest influence on the structural response, followed by cohesion and the earth pressure coefficient K_0.

9.9 Discussion

The case study illustrates the application of the Response Surface Method to the reliability analysis of a tunnel lining system.
The main conclusions are:

- the RSM provides an accurate surrogate model of the FEM response
- the method significantly reduces computational cost
- soil stiffness variability has the strongest influence on lining forces

- probabilistic methods provide a more transparent assessment of structural safety compared to deterministic approaches.

The results demonstrate that the Response Surface Method provides an efficient approach for performing reliability analysis in problems involving computationally expensive numerical models.
The use of RSM reduced the required number of finite element simulations from several tens of thousands (required for direct Monte Carlo simulation) to only **25 numerical analyses**.
This approach allows probabilistic analysis to be integrated into routine geotechnical design while maintaining reasonable computational effort.
The results demonstrate that reliability-based analysis can provide valuable insights into the performance of tunnel lining systems.
In particular, probabilistic methods allow engineers to:

- explicitly quantify structural safety levels
- identify critical geotechnical parameters
- optimize structural dimensions and reinforcement.

For large underground infrastructure projects, such as metro tunnels or railway tunnels, these methods can support improved risk management and more rational allocation of safety margins.

10. Conclusions

This paper has discussed the application of reliability-based design concepts to large geotechnical structures, with particular reference to tunnel linings and retaining systems.
The study highlights the following key points:

- geotechnical design is strongly influenced by soil variability and model uncertainty
- Eurocode 7 addresses uncertainty through partial safety factors but does not explicitly quantify structural reliability
- reliability-based approaches allow direct estimation of failure probability and reliability index
- probabilistic analyses can support improved design optimization and risk management in large geotechnical projects.

Future developments in geotechnical reliability are expected to focus on improved statistical characterization of ground properties, integration with numerical modelling, and incorporation into practical design frameworks.

References

1. **Chiarelli, M., 2009**. L'Arte del costruire gallerie – Editrice | Uni Service, Trento, 2009 ISBN: 978-88-6178-323-2
2. **Chiarelli, M., 2025.** Innovazioni tecnologiche nella stabilizzazione dei pendii: modelli costitutivi e simulazioni numeriche - INGENIO del 01 Luglio 2025, Imready Srl – RSM
3. **Chiarelli, M., 2025.** L'impiego della geofisica e del remote sensing nel monitoraggio dei fenomeni franosi: stato dell'arte, sviluppi recenti e prospettive future - INGENIO del 18 Settembre 2025, Imready Srl – RSM
4. **Chiarelli, M., 2019.** Tunnel esistenti: vulnerabilità sismica, monitoraggio delle strutture, piano sul ciclo di vita degli interventi" - INGENIO del 16 ottobre 2019, Imready Srl – RSM
5. **Chiarelli, M., 2020**. Tunnel realizzati in subalveo. Il caso del tunnel sotto lo Stretto di Messina - INGENIO del 06 Ottobre 2020, Imready Srl – RSM
6. **Allen, D.E., Nowak, A.S., 1994**. Calibration of partial safety factors for structural reliability. Journal of Structural Engineering, 120(5), pp.1405–1424.
7. **Ang, A.H.S., Tang, W.H., 2007**. Probability Concepts in Engineering: Emphasis on Applications to Civil and Environmental Engineering. 2nd ed. Wiley, New York.
8. **Baecher, G.B., Christian, J.T., 2003**. Reliability and Statistics in Geotechnical Engineering. Wiley, Chichester.
9. **Baecher, G.B., Christian, J.T., 2011**. Reliability and Statistics in Geotechnical Engineering. 2nd ed. Wiley.
10. **Brinkgreve, R.B.J., Kumarswamy, S., Swolfs, W., 2016**. PLAXIS Finite Element Code for Soil and Rock Analyses. Balkema, Rotterdam.
11. **Christian, J.T., Ladd, C.C., Baecher, G.B., 1994**. Reliability applied to slope stability analysis. Journal of Geotechnical Engineering, ASCE, 120(12), pp.2180–2207.
12. **Duncan, J.M., 2000**. Factors of safety and reliability in geotechnical engineering. Journal of Geotechnical and Geoenvironmental Engineering, 126(4), pp.307–316.
13. **El-Ramly, H., Morgenstern, N.R., Cruden, D.M., 2002**. Probabilistic slope stability analysis for practice. Canadian Geotechnical Journal, 39(3), pp.665–683.
14. **EN 1997-1, 2004. Eurocode 7**: Geotechnical Design – Part 1: General Rules. European Committee for Standardization (CEN), Brussels.
15. **EN 1990, 2002**. Eurocode: Basis of Structural Design. CEN, Brussels.
16. **Fenton, G.A., Griffiths, D.V., 2008.** Risk Assessment in Geotechnical Engineering. Wiley, New York.
17. **Griffiths, D.V., Fenton, G.A., 2004.** Probabilistic methods in geotechnical engineering. In: Proc. International Conference on Applications of Statistics and Probability in Civil Engineering.
18. **Hasofer, A.M., Lind, N.C., 1974.** Exact and invariant second-moment code format. Journal of the Engineering Mechanics Division, ASCE, 100(1), pp.111–121.
19. **Honjo, Y., 2011.** Challenges in geotechnical reliability based design. Geotechnical Engineering Journal of the SEAGS & AGSSEA, 42(1), pp.9–16.

20. **Low, B.K., Tang, W.H., 2007.** Efficient spreadsheet algorithm for first-order reliability method. Journal of Engineering Mechanics, 133(12), pp.1378–1387.

21. **Phoon, K.K., Kulhawy, F.H., 1999.** Characterization of geotechnical variability. Canadian Geotechnical Journal, 36(4), pp.612–624.

22. **Phoon, K.K., Ching, J., 2015.** Risk and Reliability in Geotechnical Engineering. CRC Press.

23. **Rackwitz, R., Fiessler, B., 1978.** Structural reliability under combined random load sequences. Computers & Structures, 9(5), pp.489–494.

24. **Schweckendiek, T., 2014.** Geotechnical Safety and Reliability. Wiley.

25. **Tang, W.H., 1971.** A probabilistic approach to design of earth slopes. Canadian Geotechnical Journal, 8(3), pp.275–287.

26. **Terzaghi, K., Peck, R.B., Mesri, G., 1996.** Soil Mechanics in Engineering Practice. 3rd ed. Wiley.

27. **Einstein, H.H., Schwartz, C.W., 1979.** Simplified analysis for tunnel supports. Journal of the Geotechnical Engineering Division, ASCE.

28. **Oreste, P., 2005.** A probabilistic design approach for tunnel supports. International Journal of Rock Mechanics and Mining Sciences.

29. **Lee, K.M., Hou, X., Ge, X., Tang, Y., 2001.** An analytical solution for tunnelling-induced ground movements. International Journal for Numerical and Analytical Methods in Geomechanics.

30. **Loganathan, N., Poulos, H.G., 1998.** Analytical prediction for tunnelling-induced ground movements. Journal of Geotechnical and Geoenvironmental Engineering.

31. **Do, N.A., Dias, D., Oreste, P., Djeran-Maigre, I., 2014.** A new numerical approach to the hyperstatic reaction method for segmental tunnel linings. Tunnelling and Underground Space Technology.